Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 2, 1998

ON THE STABILITY OF SHOCK WAVES IN A CONTINUOUS MEDIUM WITH A SPACE CHARGE

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UDC 533.6.011.72:537.8

The stability of shock waves is discussed for a hydrodynamic model of motion of a continuous medium with a space electrical charge. The correctness of a mixed problem obtained by linearization of the hydrodynamic model and the equations of a strong discontinuity for electrohydrodynamic shock waves is proved. As is known, this indicates stability of this type of strong discontinuity in the model of a continuous medium considered.

Introduction. The motion of a continuous medium with a space electrical charge has aroused steady interest in connection with various practical applications [1]. The problem of construction and substantiation of the basic electrohydrodynamic (EHD) equations is far from being adequately solved, in contrast to, say, problems of magnetohydrodynamics.

In the present paper, the system of EHD equations adopted as the basis in [1] is discussed in the context of the theory of equations with partial derivatives. In comparison to the system described in [2], it has a number of advantages from the viewpoint of the theory of differential equations. This is very important, e.g., in substantiation of numerical methods of solution of specific EHD problems.

In the present paper, the correctness of a mixed problem obtained by linearization of the EHD equations and the nonlinear relations for electrohydrodynamic shock waves is proved. This means that this type of strong discontinuity is stable in the model of a continuous medium considered.

1. Basic EHD Equations. The EHD equations in a one-liquid approximation have the form [1, 2]

$$\rho_t + \operatorname{div}(\rho \mathbf{u}) = 0; \tag{1.1}$$

$$(\rho \mathbf{u})_t + \operatorname{div}\tilde{\Pi} = 0; \tag{1.2}$$

$$(\rho e)_t + \operatorname{div}\mathbf{W} = (\mathbf{J}, \mathbf{E}); \tag{1.3}$$

$$q_t + \operatorname{div}\mathbf{J} = 0; \tag{1.4}$$

$$\operatorname{div}\mathbf{E} = 4\pi q; \tag{1.5}$$

$$\operatorname{rot}\mathbf{E} = 0. \tag{1.6}$$

Here ρ is the density of a continuous medium, $\mathbf{u} = (u_1, u_2, u_3)^*$ is the velocity of a continuous medium (the asterisk denotes transposition), $\tilde{\Pi}$ is the momentum flux density tensor with the components $\tilde{\Pi}_{ik} = \rho u_i u_k + p\delta_{ik} - P_{ik}$ $(i, k = 1, 2, 3)$, p is the pressure, $\mathbf{E} = (E_1, E_2, E_3)^*$ is the electric-field strength, $e = e_0 + (1/2)|\mathbf{u}|^2$, e_0 is the internal energy, $\mathbf{W} = (W_1, W_2, W_3)^* = \rho\mathbf{u}(e + pV)$, $V = 1/\rho$ is the specific volume, $\mathbf{J}$ is the current density, and q is the charge. The components P_{ik} of the Maxwell stress tensor P have the form $P_{ik} = (1/4\pi)(E_iE_k - |\mathbf{E}|^2\delta_{ik}/2)$. The thermodynamic variables are related by the Gibbs relation

$$T ds = de_0 + p dV \tag{1.7}$$

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(s is the entropy and T is the temperature). By virtue of (1.7), the equalities

$$p = -(e_0)_V = \rho^2(e_0)_\rho, \qquad T = (e_0)_s \tag{1.8}$$

are valid.

The current density $\mathbf{J}$ is related to the velocity $\mathbf{u}$ and electric-field strength $\mathbf{E}$ by the Ohm law

$$\mathbf{J} = q(\mathbf{u} + b\mathbf{E}) \tag{1.9}$$

(the constant $b > 0$ is the mobility [1, 2]). Thus, with allowance for the equation of state $e_0 = e_0(\rho, s)$, equalities (1.8), and the Ohm law (1.9), Eqs. (1.1)–(1.6) can be regarded as a system for determining the components of the vectors $\mathbf{U} = (p, s, \mathbf{u}^*)^*$ and $\mathbf{E}$ and the charge q. In this case, the Maxwell equations (1.5) and (1.6) can be reduced to one Poisson equation for the scalar electric potential φ ($\mathbf{E} = -\nabla\varphi$):

$$\Delta\varphi = -4\pi q. \tag{1.10}$$

By virtue of (1.5) and (1.6), the vectorial equation (1.2) can be written as

$$(\rho\mathbf{u})_t + \operatorname{div}\Pi = q\mathbf{E}, \tag{1.2$'$}$$

where Π is the momentum flux density tensor with components $\Pi_{ik} = \rho u_i u_k + p\delta_{ik}$). Then (1.1), (1.2$'$), (1.3) is a system of gas-dynamic equations with right members which can be written in nondivergent form:

$$\frac{1}{\rho c^2}\frac{dp}{dt} + \operatorname{div}\mathbf{u} = b\frac{(e_0)_{\rho s} q}{c^2 T}|\mathbf{E}|^2, \qquad \frac{ds}{dt} = b\frac{q}{\rho T}|\mathbf{E}|^2, \qquad \rho\frac{d\mathbf{u}}{dt} + \nabla p = q\mathbf{E}, \tag{1.11}$$

where $d/dt = \partial/\partial t + (\mathbf{u}, \nabla)$ and $c = \sqrt{(\rho^2(e_0)_\rho)_\rho}$ is the speed of sound in a gas [3]. System (1.11) can be written in symmetric form. Under the assumption that the thermodynamic quantities satisfy the inequalities $\rho > 0$ and $(\rho^2(e_0)_\rho)_\rho > 0$, the system will be symmetric t-hyperbolic (after Friedrichs) [4, 5]. Below we study the case of a polytropic gas [3, 4].

2. Equations of a Strong Discontinuity. We consider piecewise-smooth solutions of system (1.1)–(1.6), in which smooth pieces are separated from each other by the surface of a strong discontinuity [3, 6], described by the equation

$$\tilde{f}(t), \mathbf{x}) = f(t, \mathbf{x}') - x_1 = 0 \quad [\mathbf{x} = (x_1, \mathbf{x}'), \quad \mathbf{x}' = (x_2, x_3)]. \tag{2.1}$$

Following [1, 3, 6], we write conditions on the strong-discontinuity surface for the EHD system (1.1)–(1.6):

$$f_t[\rho] - [\rho u_1] + f_{x_2}[\rho u_2] + f_{x_3}[\rho u_3] = 0, \quad f_t[\rho u_i] - [\tilde{\Pi}_{1i}] + f_{x_2}[\tilde{\Pi}_{2i}] + f_{x_3}[\tilde{\Pi}_{3i}] = 0 \quad (i = 1, 2, 3),$$

$$f_t[\rho e] - [W_1] + f_{x_2}[W_2] + f_{x_3}[W_3] = 0, \quad [J_N] = \frac{\partial\sigma}{\partial t}, \tag{2.2}$$

$$[E_N] = -4\pi\sigma, \quad [E_k] + f_{x_k}[E_1] = 0 \quad (k = 2,\ 3).$$

Here we used the notation of [4]. In the derivation of these relations, it was assumed that a surface charge $\sigma = \sigma(t, x')$ can exist on the surface (2.1). According to the recommendations in [1, 6], we ignored the surface current strength.

Remark 2.1. For shock waves, i.e., for $j \neq 0$ and $[\rho] \neq 0$ [$j = \rho(u_N - D_N)$, $u_N = (\mathbf{u}, \mathbf{N})$, $D_N = -f_t/|\nabla\tilde{f}|$], the system of strong-discontinuity relations is a closed system for a given value of σ. From the flow parameters before the discontinuity and the value of σ it is possible to determine the flow parameters behind the discontinuity front.

3. Formulation of the Basic Problem of the Stability of Electrohydrodynamic Shock Waves. We linearize the EHD equations (1.1)–(1.6) and the strong-discontinuity relations with respect to the basic piecewise-constant solution:
for $x_1 < 0$,

$$\mathbf{U}(t, \mathbf{x}) = \hat{\mathbf{U}}_\infty = (\hat{p}_\infty, \hat{s}_\infty, \hat{u}_{1\infty}, 0, 0)^*, \quad \mathbf{E}(t, \mathbf{x}) = \hat{\mathbf{E}}_\infty = (\hat{E}_{1\infty}, 0, 0)^*, \quad q(t, \mathbf{x}) = 0;$$

and for $x_1 > 0$,

$$\mathbf{U}(t,\mathbf{x}) = \hat{\mathbf{U}} = (\hat{p},\hat{s},\hat{u}_1,0,0)^*, \quad \mathbf{E}(t,\mathbf{x}) = \hat{\mathbf{E}} = (\hat{E}_1,0,0)^*, \quad q(t,\mathbf{x}) = 0.$$

This solution satisfies conditions (2.2) if the discontinuity front is motionless and is described by the equation $x_1 = 0$, i.e., for $x_1 = 0$ the following relations hold:

$$[\hat{j}\,] = 0, \quad \left[\hat{\rho}\hat{u}_1^2 + \hat{p} - \frac{\hat{E}_1^2}{8\pi}\right] = 0, \quad \left[\hat{\rho}\hat{u}_1\left(\hat{e}_0 + \frac{1}{2}\hat{u}_1^2 + \hat{p}\hat{V}\right)\right] = 0, \quad [\hat{E}_1] = 4\pi\hat{\sigma}. \tag{3.1}$$

Here $\hat{p}_\infty = \hat{\rho}_\infty^2(e_0)_\rho(\hat{\rho}_\infty,\hat{s}_\infty)$, $\hat{T}_\infty = (e_0)_s(\hat{\rho}_\infty,\hat{s}_\infty)$, $\hat{p} = \hat{\rho}^2(e_0)_\rho(\hat{\rho},\hat{s})$, $\hat{T} = (e_0)_s(\hat{\rho},\hat{s})$, $\hat{e}_0 = e_0(\hat{\rho},\hat{s})$, $\hat{j} = \hat{\rho}\hat{u}_1$, $\hat{\rho}$, $\hat{\rho}_\infty$, $\hat{s}$, $\hat{s}_\infty$, $\hat{u}_1$, $\hat{u}_{1\infty}$, $\hat{E}_1$, and $\hat{E}_{1\infty}$ are some constants, and $\hat{\sigma} = \text{const}$ is the magnitude of the surface charge. In addition, it is assumed that the stationary discontinuity (3.1) is a shock wave, i.e., $\hat{u}_1 \neq 0$, $\hat{u}_{1\infty} \neq 0$, and $[\hat{\rho}] \neq 0$. After linearization we obtain the basic mixed problem of the stability of electrohydrodynamic shock waves.

Basic Problem. *In the region $t > 0$, $\mathbf{x} \in R_+^3$, we seek a solution of the system*

$$\begin{gathered} Lp + \operatorname{div}\mathbf{u} = \frac{\gamma-1}{\gamma}q, \quad Ls = \frac{\gamma-1}{\gamma}q, \quad \mathrm{M}^2 L\mathbf{u} + \nabla p = \hat{\mathbf{a}}q, \\ Lq + \hat{\omega}_1\xi_1 q = 0, \qquad \operatorname{div}\mathbf{E} = 4\pi q, \qquad \operatorname{rot}\mathbf{E} = 0, \end{gathered} \tag{3.2}$$

and in the region $t > 0$, $\mathbf{x} \in R_-^3$, we seek a solution of the system

$$\begin{gathered} L_\infty p + \operatorname{div}\mathbf{u} = \frac{\gamma-1}{\gamma}q, \quad L_\infty s = \frac{\gamma-1}{\gamma}q, \quad \mathrm{M}_\infty^2 L_\infty\mathbf{u} + \nabla p = \hat{\mathbf{a}}_\infty q, \\ L_\infty q + \hat{\omega}_{1\infty}\xi_1 q = 0, \quad \operatorname{div}\mathbf{E} = 4\pi q, \quad \operatorname{rot}\mathbf{E} = 0. \end{gathered} \tag{3.3}$$

For $x_1 = 0$, the solutions of both systems should satisfy the boundary conditions

$$\begin{gathered} u_1 + dp + d_0E_{1\infty} + d_1p_\infty + d_2u_{1\infty} + d_3s_\infty + d_4\Omega = 0, \quad u_k = \hat{\lambda}F_{x_k} + \hat{d}_0E_{k\infty} + \bar{v}u_{k\infty} \quad (k = 2,\ 3), \\ F_t = \mu p + \mu_0E_{1\infty} + \mu_1p_\infty + \mu_2u_{1\infty} + \mu_3s_\infty + \mu_4\Omega, \\ s = \nu p + \nu_0E_{1\infty} + \nu_1p_\infty + \nu_2u_{1\infty} + \nu_3s_\infty + \nu_4\Omega, \\ q = \theta_1q_\infty + \theta_2\Omega_t, \quad E_1 - \hat{d}E_{1\infty} = 4\pi\Omega, \quad E_k - \hat{d}E_{k\infty} = -\hat{\chi}F_{x_k} \quad (k = 2,\ 3), \end{gathered} \tag{3.4}$$

and for $t = 0$, they should satisfy the initial data

$$\begin{gathered} \mathbf{U}|_{t=0} = \mathbf{U}_0(\mathbf{x}), \quad \mathbf{E}|_{t=0} = \mathbf{E}_0(\mathbf{x}), \quad q|_{t=0} = q_0(\mathbf{x}), \quad \mathbf{x} \in R_\pm^3, \\ F\Big|_{t=0} = F_0(\mathbf{x}'), \quad \Omega\Big|_{t=0} = \Omega_0(\mathbf{x}'), \quad \mathbf{x}' \in R^2. \end{gathered} \tag{3.5}$$

Here

$$R_\pm^3 = \{\mathbf{x} \mid x^1 \gtrless 0, \mathbf{x}' \in R^2\}; \quad L = \tau + \xi_1; \quad L_\infty = æ\tau + \xi_1; \quad \tau = \frac{\partial}{\partial t}; \quad \nabla = (\xi_1,\xi_2,\xi_3)^*;$$

$$\xi_k = \frac{\partial}{\partial x_k} \quad (k = 1,\ 2,\ 3); \quad æ = \frac{\hat{u}_1}{\hat{u}_{1\infty}}; \quad \mathrm{M}^2 = \frac{\hat{u}_1^2}{\hat{c}^2}; \quad \hat{c}^2 = \gamma\hat{p}\hat{V}; \quad \hat{V} = \frac{1}{\hat{\rho}}; \quad \mathrm{M}_\infty^2 = \frac{\hat{u}_{1\infty}^2}{\hat{c}_\infty^2};$$

$$\hat{c}_\infty^2 = \gamma\hat{p}_\infty\hat{V}_\infty; \quad \hat{V}_\infty = \frac{1}{\hat{\rho}_\infty}; \quad \hat{\mathbf{a}} = \left(\frac{1}{\gamma\hat{\omega}_1},0,0\right)^*; \quad \hat{\mathbf{a}}_\infty = \left(\frac{1}{\gamma\hat{\omega}_{1\infty}},0,0\right)^*;$$

$$\hat{\omega}_1 = \frac{b\hat{E}_1}{\hat{u}_1}; \quad \hat{\omega}_{1\infty} = \frac{b_\infty\hat{E}_{1\infty}}{\hat{u}_{1\infty}}.$$

The constant $b_\infty > 0$ is the mobility for $x_1 < 0$ (it is assumed that the mobility is different on both sides of the discontinuity). Systems (3.2) and (3.3) are written in dimensionless form. Boundary conditions (3.4) are obtained by linearization of relations (2.2) and are written in dimensionless form; the coefficients in the boundary conditions can easily be written. During solution of the basic problem (3.2)–(3.5), we also obtain the functions $F = F(t,\mathbf{x}')$, a small displacement of the discontinuity front, and $\Omega = \Omega(t,\mathbf{x}')$, a small perturbation of the surface charge. Two relations from boundary conditions (3.4) should be regarded as equations for determining F and Ω. Without the last two equations systems (3.2) and (3.3) can be written as

$$A^{(0)}\mathbf{V}_t + \sum_{k=1}^{3} A^{(k)}\mathbf{V}_{x_k} + A^{(4)}\mathbf{V} = 0; \tag{3.6}$$

$$A_\infty^{(0)}\mathbf{V}_t + \sum_{k=1}^{3} A_\infty^{(k)}\mathbf{V}_{x_k} + A_\infty^{(4)}\mathbf{V} = 0, \tag{3.7}$$

where $\mathbf{V} = (\mathbf{U}, q)^*$ and matrices $A^{(k)}$ and $A_\infty^{(k)}$ $(k = \overline{0,4})$ can easily be derived.

Remark 3.1. By virtue of (1.10), the last two relations in systems (3.2) and (3.3) (the Maxwell equations) reduce to one Poisson equation for a small perturbation of the potential φ:

$$\Delta\varphi = -4\pi q, \quad \mathbf{x} \in R^3_\pm, \quad t > 0, \tag{3.8}$$

where φ is a dimensionless quantity. The boundary conditions for (3.8) have the form

$$\frac{\partial\varphi}{\partial x_1} - \hat{d}\frac{\partial\varphi_\infty}{\partial x_1} = -4\pi\Omega, \quad \varphi - \hat{d}\varphi_\infty = \hat{\chi}F. \tag{3.9}$$

Thus, to determine the potential φ we have the diffraction problem (3.8) and (3.9) [7].

4. Investigation of Conditions (3.1) on a Stationary Discontinuity. Let a stationary discontinuity that satisfies conditions (3.1) be a shock wave ($\hat{u}_1, \hat{u}_{1\infty} \neq 0$, and $[\hat{\rho}] \neq 0$). We write the second and third relations of (3.1) as

$$\bar{p} = -\gamma \mathrm{M}^2\bar{v} + \tilde{\Delta}, \qquad 1 - \bar{p}\bar{v} + \frac{\gamma-1}{2}\mathrm{M}^2(1-\bar{v}^2) = 0, \tag{4.1}$$

where $\tilde{\Delta} = 1 + \gamma\mathrm{M}^2(1-\hat{e})$, $\bar{p} = \hat{p}_\infty/\hat{p}$, and $\bar{v} = 1/æ$). Then we assume that $\hat{e}$ is a small parameter ($|\hat{e}| \ll 1$) and the conditions

$$\hat{s} > \hat{s}_\infty, \quad \hat{p} > \hat{p}_\infty > 0, \quad \hat{\rho} > \hat{\rho}_\infty > 0, \quad \hat{u}_{1\infty} > \hat{u}_1 > 0 \tag{4.2}$$

are satisfied. For $\hat{e} = 0$, these are the evolution conditions for shock waves in ordinary gas dynamics [3, 6, 8] (the evolution of electrohydrodynamic shock waves in the general case, i.e., for nonzero $\hat{e}$, is investigated in [1]). Since a polytropic gas is considered, in view of the first relation of (3.1), inequalities (4.2) can be written as

$$\bar{p}\bar{v}^\gamma < 1, \quad 0 < \bar{p} < 1, \quad \bar{v} > 1. \tag{4.3}$$

From (4.1) we obtain a quadratic equation for determining $\bar{v}$ and out of the two roots of this equation we choose the root

$$\bar{v} = \frac{\gamma}{\gamma+1}\left\{\Delta + \sqrt{\Delta^2 - 2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}\right\} \quad \left(\Delta = l + 1 - \hat{e}, \quad l = \frac{1}{\gamma\mathrm{M}^2}\right),$$

which is less than unity for $\hat{e} = 0$ (the other root is equal to unity for $\hat{e} = 0$). We obtain the following expression for $\bar{p}$:

$$\bar{p} = \frac{\gamma\mathrm{M}^2}{\gamma+1}\left\{\Delta - \gamma\sqrt{\Delta^2 - 2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}\right\}.$$

For $|\hat{e}| \ll 1$, the evolution conditions (4.3) for electrohydrodynamic shock waves impose the following restrictions on the main-flow parameters:

$$\frac{\gamma-1}{2\gamma} < \mathrm{M}^2 < 1, \quad \mathrm{M}^2_\infty = \frac{\mathrm{M}^2\bar{v}}{\bar{p}} > 1. \tag{4.4}$$

Remark 4.1. For the small parameter $\hat{e}$, the coefficients of boundary conditions (3.4) can be represented as $d = d^{(0)} + O(\hat{e})$, $\hat{d}_0 = O(\hat{e})$, $d_0 = O(\hat{e})$, $\hat{\lambda} = \hat{\lambda}^{(0)} + O(\hat{e})$, $\mu = \mu^{(0)} + O(\hat{e})$, $\mu_0 = O(\hat{e})$,

$\nu = \nu^{(0)} + O(\hat{e})$, $\nu_1 = O(\hat{e})$, and $\hat{\chi} = O(\hat{e})$, where $d^{(0)}$, $\hat{\lambda}^{(0)}$, $\mu^{(0)}$, and $\nu^{(0)}$ are some constant numbers. It is assumed that the condition of smallness of $\hat{e}$ ($|\hat{e}| \ll 1$) is satisfied by virtue of the smallness of the jump of the normal component of the electric-field strength on the discontinuity, i.e.,

$$\left|\frac{\hat{\sigma}}{\hat{u}_1\sqrt{\hat{\rho}/2\pi}}\right| = \left|\frac{\hat{E}_1}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} - \frac{\hat{E}_{1\infty}}{\hat{u}_1\sqrt{8\pi\hat{\rho}}}\right| \ll \left|\frac{\hat{E}_1}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} + \frac{\hat{E}_{1\infty}}{\hat{u}_1\sqrt{8\pi\hat{\rho}}}\right|^{-1},$$

$\operatorname{sgn}\hat{E}_1 = \operatorname{sgn}\hat{E}_{1\infty}$, and the quantities $\hat{E}_1/(\hat{u}_1\sqrt{8\pi\hat{\rho}})$ and $\hat{E}_{1\infty}/(\hat{u}_1\sqrt{8\pi\hat{\rho}}\ /)$ are not small.

Remark 4.2. Taking into account the specificity of the stationary discontinuity (3.1), one obtains several variants of the basic problem. In the formulation of mixed problems for systems (3.6) and (3.7), it is necessary to know the eigenvalues of the matrices $A^{(1)}$ and $A^{(1)}_\infty$. The eigenvalues of the matrix $A^{(1)}$ have the form

$$\lambda_1 = 1, \quad \lambda_{2,3} = \mathrm{M}^2, \quad \lambda_4 = 1 + \hat{\omega}_1, \quad \lambda_{5,6} = \frac{1 + \mathrm{M}^2 \pm \sqrt{(1+\mathrm{M}^2)^2 + 4(1-\mathrm{M}^2)}}{2}. \tag{4.5}$$

The matrix $A^{(1)}_\infty$ has eigenvalues of a similar form. By virtue of (4.4), we have $\lambda_{1,2,3,5}(A^{(1)}) > 0$ and $\lambda_{1,2,3,5,6}(A^{(1)}_\infty) > 0$.

Remark 4.3. Let the conditions

$$1 + \hat{\omega}_{1\infty} > 0, \quad 1 + \hat{\omega}_1 > 0, \tag{4.6}$$

or

$$1 + \hat{\omega}_{1\infty} < 0, \quad 1 + \hat{\omega}_1 < 0, \tag{4.7}$$

or

$$1 + \hat{\omega}_{1\infty} > 0, \quad 1 + \hat{\omega}_1 < 0 \tag{4.8}$$

be satisfied. With satisfaction of (4.6) all eigenvalues of the matrix $A^{(1)}_\infty$ are positive, i.e., for system (3.7) one need not set boundary conditions for $x_1 = 0$. At the same time, by virtue of (4.5), system (3.6) requires five boundary conditions. Thus, to pose the basic problem correctly from the viewpoint of the number of boundary conditions with satisfaction of inequalities (4.6), it is necessary to establish satisfaction of the identity

$$\Omega \equiv 0. \tag{4.9}$$

Otherwise, the basic problem will be underdetermined in the number of boundary conditions. Similarly, in the case of satisfaction of conditions (4.7), the basic problem is correctly posed in the number of boundary conditions if identity (4.9) is satisfied. When conditions (4.8) are satisfied, it is correctly posed if $\Omega \not\equiv 0$. Note that, with satisfaction of the conditions $1 + \hat{\omega}_{1\infty} < 0$ and $1 + \hat{\omega}_1 > 0$, the basic problem is underdetermined even for $\Omega \equiv 0$.

Remark 4.4. The physical meaning of conditions (4.6) is that with satisfaction of these conditions, by virtue of the Ohm law (1.9), electric current flows downstream of the discontinuity from left to right [in the case of satisfaction of conditions (4.7), electric current flows upstream of the discontinuity from right to left]. With satisfaction of conditions (4.8), electric current is directed to the discontinuity from both sides, generating a surface charge on the shock wave.

5. Investigation of the Correctness of the Basic Problem. We describe the process of deriving an *a priori* estimate without loss of smoothness for the solution of the basic problem in the case of satisfaction of conditions (4.6). For this, we construct extended systems for systems (3.6) and (3.7) [4]. The process of deriving these systems consists of two stages. In the first stage, from (3.6) and (3.7) we construct extended systems (for determining the components of the vector $\mathbf{V}$ and its derivatives) and write for them identities of the energy integrals in differential form [4]. Integrating the identities over the regions R^3_+ and R^3_-, respectively, and combining the resulting expressions, we obtain the equality

$$\frac{d}{dt}I_0(t) - \iint_{R^2}[(A_p^{(1)}\mathbf{V}_p, \mathbf{V}_p)]\Big|_{x_1=0} dx' + \iiint_{R^3_+}((A_p^{(4)} + A_p^{(4)*})\mathbf{V}_p, \mathbf{V}_p)\, dx$$

$$+ \iiint_{R^3_-}((A_{p\infty}^{(4)} + A_{p\infty}^{(4)*})\mathbf{V}_p, \mathbf{V}_p)\, dx = 0, \tag{5.1}$$

where

$$I_0(t) = \iiint_{R^3_+}(A_p^{(0)}\mathbf{V}_p, \mathbf{V}_p)\, dx + \iiint_{R^3_-}(A_{p\infty}^{(0)}\mathbf{V}_p, \mathbf{V}_p)\, dx,$$

$$\mathbf{V}_p = (\mathbf{V}^*; \tau\mathbf{V}^*; \xi_1\mathbf{V}^*; \xi_2\mathbf{V}^*; \xi_3\mathbf{V}^*; \tau^2\mathbf{V}^*; \tau\xi_1\mathbf{V}^*; \tau\xi_2\mathbf{V}^*; \tau\xi_3\mathbf{V}^*; \xi_1^2\mathbf{V}^*; \xi_1\xi_2\mathbf{V}^*; \xi_1\xi_3\mathbf{V}^*; \xi_2^2\mathbf{V}^*; \xi_2\xi_3\mathbf{V}^*; \xi_3^2\mathbf{V}^*)^*,$$

$A_p^{(\alpha)} = \mathrm{diag}(I_5 \times A^{(\alpha)}, \varepsilon(I_{10} \times A^{(\alpha)}))$, $A_{p\infty}^{(\alpha)} = \eta(I_{15} \times A_\infty^{(\alpha)})$ ($\alpha = \overline{0,4}$) are block diagonal matrices, $I_5 \times A^{(\alpha)}$ are the Kronecker product of the matrices I_5 and $A^{(\alpha)}$, I_5 is a unit matrix of the order of 5, etc., and ε and $\eta > 0$ are some constants. In the derivation of (5.1), it was assumed that $|\mathbf{V}_p| \to 0$ as $|x_k| \to \infty$ ($k = 1$, 2, and 3).

Estimating the second and third terms in equality (5.1) by means of boundary conditions (3.4) for $\Omega = 0$ and system (3.2) for $x_1 = 0$, by virtue of the positive definiteness of the matrix $A_{p\infty}^{(1)}$ (see Remarks 4.2 and 4.3), we obtain the inequality

$$\frac{d}{dt}I_0(t) + \eta\lambda_{\min}\iint_{R^2}(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty})\, dx' - N_1H_1(t) - N_{\hat{e}}H_2(t) \leqslant N_2I_0(t), \tag{5.2}$$

where $N_1, N_2 > 0$, and $N_{\hat{e}} = O(\hat{e})$ are constants;

$$H_1(t) = \iint_{R^2}\{p^2 + u_2^2 + u_3^2 + p_t^2 + p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + \ldots\}\Big|_{x_1=0} + \varepsilon(P + R) + (\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty})\, dx';$$

$$(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}) = (\mathbf{V}_p, \mathbf{V}_p)\Big|_{x_1=0}; \quad P = \Big(p_{tt}^2 + p_{tx_1}^2 + p_{tx_2}^2 + p_{tx_3}^2 + \sum_{i=1}^{3}\sum_{j=i}^{3} p_{x_ix_j}^2\Big)\Big|_{x_1=0};$$

$$R = \sum_{i=2}^{3}\sum_{j=2}^{3}\sum_{k=j}^{3}(u_i)_{x_jx_k}^2\Big|_{x_1=0}; \qquad H_2(t) = \iint_{R_2}\mathcal{E}\Big|_{x_1=0} dx';$$

$$\mathcal{E}\Big|_{x_1=0} = |\mathbf{E}_\infty|^2 + \left|\frac{\partial\mathbf{E}_\infty}{\partial t}\right|^2 + \left|\frac{\partial\mathbf{E}_\infty}{\partial x_2}\right|^2 + \left|\frac{\partial\mathbf{E}_\infty}{\partial x_3}\right|^2$$

$$+\varepsilon\left\{\left|\frac{\partial^2\mathbf{E}_\infty}{\partial t^2}\right|^2 + \left|\frac{\partial^2\mathbf{E}_\infty}{\partial t\partial x_2}\right|^2 + \left|\frac{\partial^2\mathbf{E}_\infty}{\partial t\partial x_3}\right|^2 + \left|\frac{\partial^2\mathbf{E}_\infty}{\partial x_2^2}\right|^2 + \left|\frac{\partial^2\mathbf{E}_\infty}{\partial x_2\partial x_3}\right|^2 + \left|\frac{\partial^2\mathbf{E}_\infty}{\partial x_3^2}\right|^2\right\}.$$

We estimate $H_2(t)$ using the integral $H_1(t)$. For this, we apply a Fourier transform to problem (3.8) and (3.9) with allowance for (4.9). As a result, we obtain the following boundary-value problem for an ordinary differential equation:

$$\frac{d^2\hat{\varphi}}{dx_1^2} - \omega^2\hat{\varphi} = -4\pi\hat{q}, \quad X^1 \gtrless 0; \tag{5.3}$$

$$\left(\frac{d\hat{\varphi}}{dx_1} - \hat{d}\frac{d\hat{\varphi}_\infty}{dx_1}\right)\Big|_{x_1=0} = 0, \quad (\hat{\varphi} - \hat{d}\hat{\varphi}_\infty)\Big|_{x_1=0} = \hat{\chi}\hat{F}. \tag{5.4}$$

Here $\hat{\varphi}$, $\hat{q}$, and $\hat{F}$ are the transforms of the Fourier functions $\varphi(t,\mathbf{x})$, $q(t,\mathbf{x})$, and $F(t,\mathbf{x}')$; $\omega^2 = 4\pi^2|\xi'|^2 = 4\pi^2(\tilde{\xi}_2^2 + \tilde{\xi}_3^2) < \infty$, where $\xi' = (\tilde{\xi}_2, \tilde{\xi}_3)$ is the parameter of the Fourier transform. Following [4], it is easy to obtain a solution of the boundary-value problem (5.3) and (5.4):

$$\hat{\varphi} = -\tilde{m}\exp(-\omega x_1)c_2 + \tilde{y}_1, \quad \hat{\varphi}' = \frac{1}{2}c_2\exp(-\omega x_1) + \omega\tilde{y}_1 + \tilde{y}_2 \quad (x_1 > 0),$$
$$\hat{\varphi} = c_{1\infty}\exp(\omega x_1) + \tilde{y}_{1\infty}, \quad \hat{\varphi}' = c_{1\infty}\omega\exp(\omega x_1) + \omega\tilde{y}_{1\infty} + \tilde{y}_{2\infty} \quad (x_1 < 0), \tag{5.5}$$

where

$$(\tilde{y}_1, \tilde{y}_2)^* = \int_0^\infty G(x_1 - \tau)\mathbf{f}(t, \tau, \xi')\,d\tau; \quad (\tilde{y}_{1\infty}, \tilde{y}_{2\infty})^* = \int_{-\infty}^0 G(x_1 - \tau)\mathbf{f}(t, \tau, \xi')\,d\tau;$$

$$\mathbf{f} = \begin{pmatrix} 0 \\ -4\pi\hat{q} \end{pmatrix}; \quad c_{1\infty} = \frac{2\pi}{\hat{d}\omega}\int_0^\infty \exp(-\omega\tau)\hat{q}(t, \tau, \xi')\,d\tau - \frac{\hat{\chi}}{2\hat{d}}\hat{F};$$

$$c_2 = -4\pi\hat{d}\int_{-\infty}^0 \exp(\omega\tau)\hat{q}(t, \tau, \xi')\,d\tau - \hat{\chi}\omega\hat{F}.$$

The function $H_2(t)$ is the sum of integrals:

$$H_2(t) = \iint_{R^2} E_{1\infty}^2 d\mathbf{x}' + \iint_{R^2} E_{2\infty}^2 d\mathbf{x}' + \ldots + \varepsilon\iint_{R^2}\left(\frac{\partial^2 E_{3\infty}}{\partial x_3^2}\right)^2 d\mathbf{x}'.$$

Using the Parseval equality, the second and third boundary conditions in (3.4), and relations (5.5), we obtain the inequality

$$\iint_{R^2} E_{1\infty}^2 d\mathbf{x}' \leqslant K_1\iint_{R^2}(u_2^2 + u_3^2)\Big|_{x_1=0} d\mathbf{x}' + K_2\iint_{R^2}(u_{2\infty}^2 + u_{3\infty}^2)\,d\mathbf{x}'$$
$$+K_3\iint_{R^2}\left\{\left|\int_0^\infty \exp(-\omega\tau)\hat{q}d\tau\right|^2 + \left|\int_{-\infty}^0 \exp(\omega\tau)\hat{q}d\tau\right|^2\right\} d\xi', \tag{5.6}$$

where $K_1, K_2, K_3 > 0$ are constants, which can easily be written.

From the last equation of system (3.6) it follows that the function

$$\Phi = \Phi(t, \xi') = \int_0^\infty \exp(-\omega\tau)\hat{q}(t, \tau, \xi')\,d\tau$$

satisfies the equation -3

$$\Phi_t + (1 + \hat{\omega}_1)\omega\Phi = \hat{q}(t, +0, \xi'), \tag{5.7}$$

where $\hat{q}(t, +0, \xi') = \hat{q}(t, \tau, \xi')\big|_{\tau\to+0} = \theta_1\hat{q}(t, \tau, \xi')\big|_{\tau\to-0} = \theta_1\hat{q}_\infty(t, \xi')$ (we assume that $q \to 0$ for $x_1 \to \infty$).

From (5.7) we obtain

$$\Phi = \exp(-(1 + \hat{\omega}_1)\omega t)\int_0^\infty \exp(-\omega\tau)\hat{q}_0(\tau, \xi')\,d\tau + \theta_1\int_0^t \exp(-(1 + \hat{\omega}_1)\omega(t - z))\hat{q}_\infty(z, \xi')\,dz. \tag{5.8}$$

On the other hand, by virtue of the first inequality of (4.6) and the last equation of system (3.7), the function $q(t, \mathbf{x})$ for $x_1 < 0$ is defined through the initial data as follows:

$$q(t, \mathbf{x}) = q_0(x_1 - \bar{v}(1 + \hat{\omega}_{1\infty})t, \mathbf{x}'), \quad x_1 < 0. \tag{5.9}$$

We assume that the function $q_0(x)$ is finite in x_1 with the supporter $\operatorname{supp} q_0 = ((z_{1\infty}, z_{0\infty}) \cup (z_0, z_1)) \times R^2$, where $-\infty < z_{1\infty} < z_{0\infty} \leqslant 0 \leqslant z_0 < z_1 < \infty$. Then, taking into account the Hölder inequality and (5.9), from (5.8) we have

$$|\Phi|^2 \leqslant C_0 \int_0^{\infty} |\hat{q}_0|^2 \, d\tau + C_1 \int_{-\infty}^{0} |\hat{q}_0|^2 \, d\tau, \tag{5.10}$$

where the constants C_0, $C_1 > 0$ depend on $z_{1\infty}$, $z_{0\infty}$, z_0, and z_1. Thus, using the Hölder inequality, from (5.6) and (5.10) we ultimately derive the estimate

$$\iint_{R^2} E_{1\infty}^2 dx' \leqslant K_1 \iint_{R^2} (u_2^2 + u_3^2)\Big|_{x_1=0} dx' + K_2 \iint_{R^2} (u_{2\infty}^2 + u_{3\infty}^2) \, dx' + K_4 I_0(t),$$

where $K_4 > 0$ is a constant. Similarly, it is possible to estimate the remaining integrals in the sum for $H_2(t)$. As a result, we obtain the estimate $H_2(t) \leqslant N_3 H_1(t) + N_4 I_0(t)$ (N_3 and $N_4 > 0$ are constants) from which, using the property of trace of the function from $W_2^1(R_+^3)$ on the plane $x_1 = 0$ [9], we derive the inequality

$$H_2(t) \leqslant N_3 \iint_{R^2} (\varepsilon(P + R) + (\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty})) \, dx' + \tilde{N}_4 I_0(t), \tag{5.11}$$

where $\tilde{N}_4 > 0$ is a constant.

From boundary conditions (3.4) and system (3.2), for $x_1 = 0$ we obtain

$$(\xi_2^2 + \xi_3^2) u_k = (\beta_1 \tau + \beta_2 \xi_1) \xi_k p - d_0 \tau \xi_k E_{1\infty} + \sum_{i=1}^{3} D_k^{(i)} u_{i\infty} + D_k^{(4)} p_\infty + D_k^{(5)} s_\infty + D_k^{(6)} q_\infty$$

$$(k = 2, 3), \quad x_1 = 0,$$

where $\beta_1 = -1 - d$, $\beta_2 = \beta^2/\mathrm{M}^2$, $\beta = \sqrt{1 - \mathrm{M}^2}$ $(\mathrm{M} < 1)$, and the differential operators $D_k^{(j)}$ $(j = \overline{1,5})$ are of the form

$$D_k^{(j)} = \sum_{\alpha_0 + |\alpha| = 2} d_{\alpha_0, \alpha}^{kj} \tau^{\alpha_0} \xi^{\alpha}, \quad D_k^{(6)} = d_1^k \tau + d_2^k \xi_2 + d_3^k \xi_3,$$

where α_0 is a nonnegative integer, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is a multiindex with nonnegative integral components, $|\alpha| = \alpha_1 + \alpha_2 + \alpha_3$, $\xi_\alpha = \xi_1^{\alpha_1} \xi_2^{\alpha_2} \xi_3^{\alpha_3}$, and the constants $d_{\alpha_0,\alpha}^{kj}$ and $d_{1,2,3}^k$ are expressed in terms of the coefficients of boundary conditions (3.4). Then, using the known inequality [10]

$$\iint_{R^2} R dx' \leqslant \text{const} \iint_{R^2} \sum_{k=2}^{3} (\xi_2^2 u_k + \xi_3^2 u_k)^2 \Big|_{x_1=0} dx'$$

$$\leqslant C_1 \iint_{R^2} \sum_{k=2}^{3} ((\beta_1 \tau + \beta_2 \xi_1) \xi_k p)^2 \Big|_{x_1=0} dx' + C_2 H_2(t) + C_3 \iint_{R^2} (\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}) \, dx'$$

($C_{1,2,3} > 0$ are constants) and the property of trace of the function from $W_2^1(R_+^3)$ on the plane $x_1 = 0$ with allowance for (5.11), we reduce inequality (5.2) to the form

$$\frac{d}{dt} I_0(t) + \iint_{R^2} ((\eta \lambda_{\min} - \tilde{N}_3)(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}) - \varepsilon \tilde{N}_1 P) \, dx' \leqslant \tilde{N}_2 I_0(t) \tag{5.12}$$

($\tilde{N}_1$, $\tilde{N}_2$, and $\tilde{N}_3 > 0$ are constants). Note that in the derivation of (5.12), $\varepsilon < 1/(N_3(1 + C_2))$. Inequality (5.11) can be written as

$$H_2(t) \leqslant \varepsilon \hat{N}_3 \iint_{R^2} P \, dx' + C_4 \iint_{R^2} (\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}) \, dx' + \hat{N}_4 I_0(t), \tag{5.11'}$$

where

$$\hat{N}_3 = \frac{N_3 C_1}{1 - \varepsilon N_3 (1 + C_2)} > 0, \quad \hat{N}_4 = \frac{\tilde{N}_4}{1 - \varepsilon N_3 (1 + C_2)} > 0,$$

and $C_4 > 0$ is a constant.

We proceed to the second stage of construction of the extended system. Note that the function p for $x_1 > 0$ satisfies the wave equation

$$\{(\tau')^2 - (\xi_1')^2 - \xi_2^2 - \xi_3^2\}p = \mathcal{F}_1 \tag{5.13}$$

$[\mathcal{F}_1 = (\mathrm{M}^2(\gamma - 1)/\gamma)Lq - 1/(\gamma\hat{\omega}_1)\xi_1 q]$, where the new differential operators τ' and ξ_1' are given by the formulas $\tau = (\beta/\mathrm{M})\tau'$ and $\xi_1 = (1/\beta)\xi_1' + (\mathrm{M}/\beta)\tau'$. If the function p satisfies Eq. (5.13), the vector $\mathbf{Y} = (\tau' p, \xi_1' p, \xi_2 p, \xi_3 p)^*$ satisfies the symmetric system [4]

$$(B\tau' + Q\xi_1' + R_2\xi_2 + R_3\xi_3)\mathbf{Y} = \mathcal{F}, \quad \mathbf{x} \in R^3_+. \tag{5.14}$$

Here $\mathcal{F} = \mathcal{F}(m_1, l_2, l_3) = (\mathcal{F}_1, -m_1\mathcal{F}_1, -l_2\mathcal{F}_1, -l_3\mathcal{F}_1)^*$, where m_1, l_2, and l_3 are constants; the matrices B, Q, R_2, and R_3 can easily be derived ($B > 0$ if $m_1^2 + l_2^2 + l_3^2 < 1$).

As in gas dynamics [4], taking into account boundary conditions (3.4), systems (3.2) and (3.3), and Eqs. (5.13), for $x_1 = 0$ we infer that the function p satisfies the boundary condition

$$(\tau' - a\xi_1')\hat{L}p + \mathcal{F}_0 = 0, \tag{5.15}$$

where $\hat{L} = a_1\tau' + a_2\xi_1'$;

$$\mathcal{F}_0 = -\frac{\mathrm{M}^2}{\beta^2}\{d_0\tau^2 - d_0\tau\xi_1 - \hat{\lambda}\mu_0(\xi_2^2 + \xi_3^2)\}E_{1\infty} + \sum_{i=1}^{3} D^{(i)}u_{i\infty} + D^{(4)}p_\infty + D^{(5)}s_\infty + D^{(6)}q_\infty;$$

$$D^{(j)} = \sum_{\alpha_0+|\alpha|=2} d^j_{\alpha_0,\alpha}\tau^{\alpha_0}\xi^\alpha, \quad j = \overline{1,5}; \quad D^{(6)} = d_{(1)}\tau + d_{(2)}\xi_2 + d_{(3)}\xi_3;$$

the constants $d^j_{\alpha_0,\alpha}$ and $d_{(1,2,3)}$ are expressed in terms of the coefficients of boundary conditions (3.4). For the vector $\mathbf{Y}_p = (\tau'\mathbf{Y}^*, \xi_1'\mathbf{Y}^*, \xi_2\mathbf{Y}^*, \xi_3\mathbf{Y}^*, \hat{L}\mathbf{Y}^*)^*$, from (5.14) we construct the extended system

$$\left\{B_p\tau' + Q_p\xi_1' + R_{2p}\xi_2 + R_{3p}\xi_3\right\}\mathbf{Y}_p = \mathcal{F}_p, \tag{5.16}$$

where B_p, Q_p, R_{2p}, and R_{3p} are block-diagonal matrices of the order of 20, $B_p = \mathrm{diag}\,(\sigma_1 B_1, \sigma_2 B_2, \sigma_3 B_3, \sigma_4 B_4, \sigma_5 B_5)$, $B_i = B(m_{1i}, l_{2i}, l_{3i})$, etc., $\sigma_i > 0$, m_{1i}, l_{2i}, and l_{3i} ($i = \overline{1,5}$) are constants, and $m_{1i}^2 + l_{2i}^2 + l_{3i}^2 < 1$. Choosing appropriate coefficients, it is possible to convert this system to a symmetric t-hyperbolic system (after Friedrichs). Furthermore, taking into account (5.15), it is possible to estimate the quadratic form as

$$-(Q_p\mathbf{Y}_p, \mathbf{Y}_p)\big|_{x_1=0} \geqslant N_4 P - \tilde{N}_{\hat{\varepsilon}}\mathcal{E} - N_5(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}), \tag{5.17}$$

where $N_{4,5}$, and $\tilde{N}_{\hat{\varepsilon}}$ are positive constants, $\tilde{N}_{\hat{\varepsilon}} = O(\hat{\varepsilon})$. For system (5.16) the energy integral is written in differential form [4]:

$$(D_p\mathbf{Y}_p, \mathbf{Y}_p)_t + \beta(Q_p\mathbf{Y}_p, \mathbf{Y}_p)_{x_1} + (R_{2p}\mathbf{Y}_p, \mathbf{Y}_p)_{x_2} + (R_{3p}\mathbf{Y}_p, \mathbf{Y}_p)_{x_3} + 2(\mathbf{Y}_p, \mathcal{F}_p) = 0. \tag{5.18}$$

Here $D_p = (\mathrm{M}/\beta)B_p - (\mathrm{M}^2/\beta)Q_p > 0$. We integrate (5.18) over the region R^3_+, assuming that $|\mathbf{Y}_p| \to 0$ for x_1, $|x_{2,3}| \to \infty$. As a result, taking into account (5.17), (5.11′), and the property of trace of the function from $W_2^1(R^3_+)$ on the plane $x_1 = 0$, we obtain the inequality

$$\frac{d}{dt}I_1(t) + \iint_{R^2}(N_6 P - N_7(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty}))\,d\mathbf{x}' \leqslant N_8(I_1(t) + I_0(t)), \tag{5.19}$$

where $N_{6,7,8}$ are positive constants, and

$$I_1(t) = \iiint_{R^3}(D_p\mathbf{Y}_p, \mathbf{Y}_p)\,d\mathbf{x}.$$

Combining inequalities (5.12) and (5.19) and taking into account that choosing appropriate constants ε and η, one can achieve positive definiteness of the quadratic form $(N_6 - \varepsilon\tilde{N}_1)P + (\eta\lambda_{\min} - N_3 - N_7)(\mathbf{V}_{p\infty}, \mathbf{V}_{p\infty})$, we obtain the inequality

$$\frac{d}{dt}I(t) \leqslant N_9 I(t), \quad t > 0,$$

where $I(t) = I_0(t) + I_1(t)$ and $N_9 > 0$ is a constant. The last inequality leads to the *a priori* estimate

$$I(t) \leqslant \exp\,(N_9 t) I(0), \quad t > 0. \tag{5.20}$$

Then, from (5.5), (5.20), and the Parseval equality we obtain the desired *a priori* estimate

$$\|\mathbf{Z}(t)\|_{W_2^2(R^3_\pm)} \leqslant N_{10}, \quad 0 < t \leqslant \tilde{T} < \infty, \tag{5.21}$$

where $\mathbf{Z} = (\mathbf{V}^*, \mathbf{E}^*)^*$; $N_{10} < \infty$ is a positive constant that depends on $\tilde{T}$; $\|\mathbf{Z}(t)\|_{W_2^2(R^3_\pm)} = \|\mathbf{Z}(t)\|_{W_2^2(R^3_+)} + \|\mathbf{Z}(t)\|_{W_2^2(R^3_-)}$. As in [4], for the function $F(t, \mathbf{x}')$ we can obtain the estimate

$$\|F\|_{W_2^3((0,\tilde{T})\times R^2)} \leqslant N_{11}, \tag{5.22}$$

where $N_{11} < \infty$ is a positive constant that depends on $\tilde{T}$.

The *a priori* estimates (5.21) and (5.22) indicate that in the case of (4.6), the basic problem of the stability of electrohydrodynamic shock waves is correct under the assumptions that on the discontinuity the jump of the normal component of the electric-field strength is small (see Remark 4.1) and the function of the initial perturbation of the charge $q_0(\mathbf{x})$ is finite in x_1 ahead of and behind the discontinuity (for $x_1 < 0$ and $x_1 > 0$).

Remark 5.1. Using the positive definiteness of the matrix $A^{(1)}_\infty$ and the procedure of deriving of an *a priori* estimate described above, it is possible to prove that the basic problem is also correct when conditions (4.7) are satisfied. When conditions (4.8) are satisfied for $-1 < \hat{\omega}_{1\infty} < 0$, i.e., $\hat{E}_{1\infty} < 0$, the condition of smallness of the coefficients $\hat{d}_0$, d_0, μ_0, and ν_0 can be satisfied (see Remark 4.1). In this case, the function $\Omega(t, x')$ is determined from the sixth boundary condition (3.4) via the initial data $q_0(\mathbf{x})$, $\mathbf{x} \in R^3_\pm$ at $x_1 = 0$. Further reasoning for deriving an *a priori* estimate for the basic problem is similar to the reasoning for the cases of (4.6) and (4.7).

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 96-01-01560).

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